Finite Math - J-term 2017

Lecture Notes - 1/24/2017

Homework

- Section 5.2 - 9, 10, 11, 12, 16, 17, 18, 19, 20, 21, 24, 33

Section 5.2 - Systems of Linear Inequalities in Two Variables

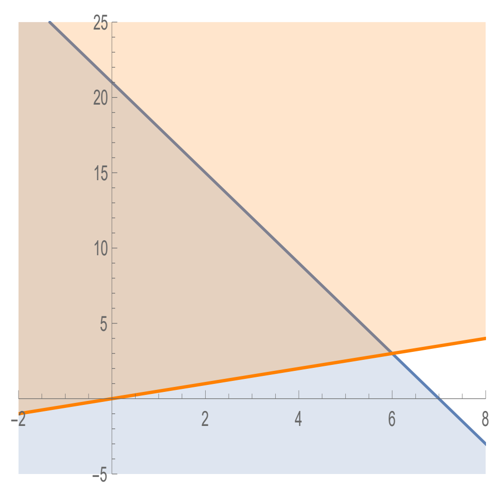
Solving Systems of Linear Inequalities Graphically.

Definition 1 (Solution Region/Feasible Region). *Given a system of inequalities, the* solution region *or* feasible region *consists of all points* (x, y) *which simultaneously satisfy all of the inequalities in the system.*

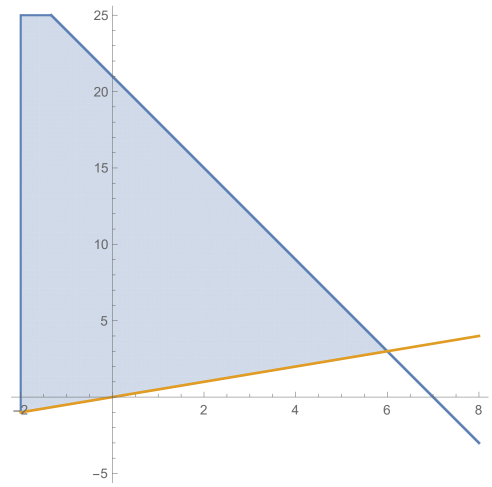
Example 1. *Solve the following system of linear inequalities graphically:*

$$\begin{array}{rcrcr} 3x & + & y & \leq & 21 \\ x & - & 2y & \leq & 0 \end{array}$$

Solution. *First we begin by graphing both inequalities on the same set of axes*



then we keep only the portion that the two graphs have in common



Example 2. *Solve the following system of linear inequalities graphically:*

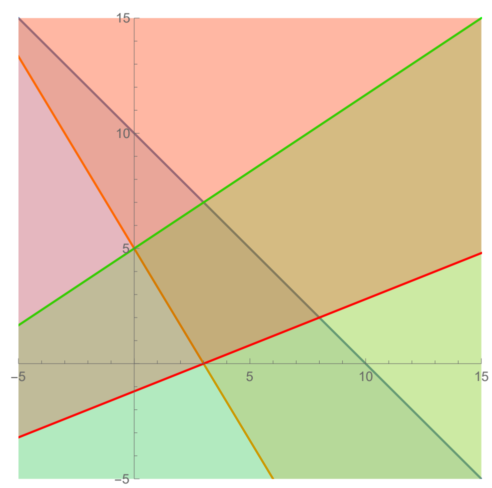
$$\begin{array}{rcrcr} 3x & + & y & \geq & 6 \\ x & - & 5y & \leq & 5 \end{array}$$

Definition 2 (Corner Point)**.** *A* corner point *of a solution region is a point in the solution region that is the intersection of two boundary lines.*

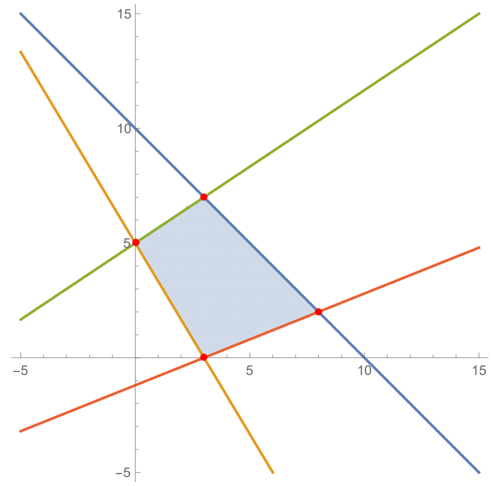
Example 3. *Solve the following system of linear inequalities graphically and find the corner points:*

$$\begin{array}{rcrcr} x & + & y & \leq & 10 \\ 5x & + & 3y & \geq & 15 \\ -2x & + & 3y & \leq & 15 \\ 2x & - & 5y & \leq & 6 \end{array}$$

Solution. *Begin by plotting all of the inequalities*



Blue is $x + y \leq 10$, orange is $5x + 3y \geq 15$, green is $-2x + 3y \leq 15$, and red is $2x - 5y \leq 6$. Then we keep only the portion that the four graphs have in common



In the above graph, the four corner points have been highlighted. To find these, we have to solve the systems of equations each intersection comes from. The intersections come from blue and green, blue and red, orange and green, and orange and red. Using the graphing method AND CHECKING THE SOLUTIONS, we can find that the corner points are

colors	*system*	*corner point*
blue and green	$\begin{cases} x+y = 10 \\ -2x+3y = 15 \end{cases}$	$(3,7)$
blue and red	$\begin{cases} x+y = 10 \\ 2x-5y = 6 \end{cases}$	$(8,2)$
orange and green	$\begin{cases} 5x+3y = 15 \\ -2x+3y = 15 \end{cases}$	$(0,5)$
orange and red	$\begin{cases} 5x+3y = 15 \\ 2x-5y = 6 \end{cases}$	$(3,0)$

Example 4. *Solve the following system of linear inequalities graphically and find the corner points:*

$$\begin{array}{rcrcl} 5x & + & y & \geq & 20 \\ x & + & y & \geq & 12 \\ x & + & 3y & \geq & 18 \\ & & x & \geq & 0 \\ & & y & \geq & 0 \end{array}$$

Definition 3 (Bounded/Unbounded)**.** *A solution region of a system of linear inequalities is* bounded *if it can be enclosed within a circle. If it cannot be enclosed within a circle, it is* unbounded*.*

Remark 1. *The solution region in Example 1 is unbounded and the solution region in Example 3 is bounded.*

Applications.

Example 5. *A manufacturing plant makes two types of inflatable boats–a two-person boat and a four-person boat. Each two-person boat requires* 0.9 *labor-hour in the cutting department and* 0.8 *labor-hour in the assembly department. Each four-person boat requires* 1.8 *labor-hours in the cutting department and* 1.2 *labor-hours in the assembly department. The maximum labor-hours available each month in the cutting and assembly departments are* 864 *and* 672*, respectively.*

(a) *Summarize this information in a table.*

(b) *If* x *two-person boats and* y *four-person boats are manufactured each month, write a system of linear inequalities that reflects the conditions indicated. Graph the feasible region.*

Solution.

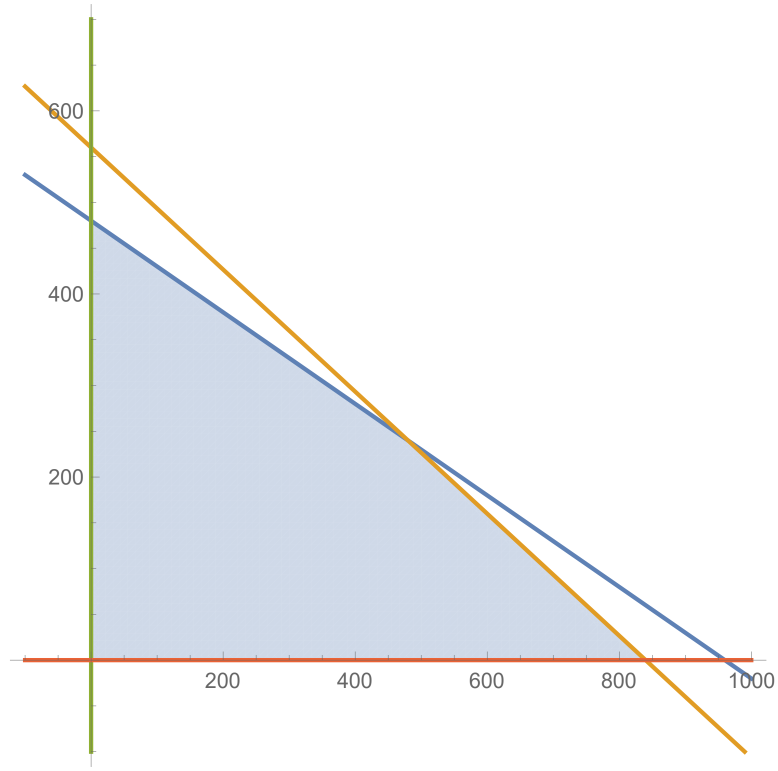
(a) *Begin by organizing the information into a table.*

	Two-Person Boat Labor-Hours	*Four-Person Boat Labor-Hours*	*Maximum Labor-Hours Available per Month*
Cutting	0.9	1.8	864
Assembly	0.8	1.2	672

(b) *The table lets us quickly come up with the system of inequalities*

$$\begin{array}{rcrcl} 0.9x & + & 1.8y & \leq & 864 \\ 0.8x & + & 1.2y & \leq & 672 \\ & & x & \geq & 0 \\ & & y & \geq & 0 \end{array}$$

Graphing all of these gives us



Example 6. *A manufacturing company makes two types of water skis, a trick ski and a slalom ski. The trick ski requires* 6 *labor-hours for fabricating and* 1 *labor-hour for finishing. The trick slalom requires* 4 *labor-hours for fabricating and* 1 *labor-hour for finishing. The maximum labor-hours available per day for fabricating and finishing are* 108 *and* 24*, respectively. If* x *is the number of trick skis and* y *is the number of slalom skis produced per day, write a system of linear inequalities that indicates appropriate restraints on* x *and* y*. Find the set of feasible solutions graphically for the number of each type of ski that can be produced.*